



Usktrimi 1: Gjenerali përpjehet e bërëndeshit kur jepet njera e elementeve të tij.

① $a=12, \beta=40^\circ, \gamma=80^\circ \parallel b=? c=? \alpha=? S=? R=?$



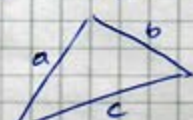
$\alpha = 180^\circ - (\gamma + \beta) = 180^\circ - 120^\circ = 60^\circ$
 $b = \frac{12 \cdot \sin 40^\circ}{\sin 60^\circ} \approx 8,9$
 $c = \frac{12 \cdot \sin 80^\circ}{\sin 60^\circ} \approx 13,6$
 $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} = 2R$
 $\frac{12}{\sin 60^\circ} = \frac{b}{\sin 40^\circ} = \frac{c}{\sin 80^\circ} = 2R$
 $2R = \frac{12}{\frac{\sqrt{3}}{2}} = \frac{24}{\sqrt{3}} = \frac{24\sqrt{3}}{3} = 8\sqrt{3} \Rightarrow R = 4\sqrt{3} \approx 6,9$
 $S = \frac{1}{2} b \cdot c \cdot \frac{\sqrt{3}}{2} \approx 52,4$

② $a=4, b=7, \gamma=30^\circ \parallel \alpha=? \beta=? c=? S=? R=?$




$c^2 = a^2 + b^2 - 2ab \cos \gamma = 16 + 49 - 2 \cdot 28 \cdot \frac{\sqrt{6}}{2} = 65 - 28\sqrt{6} \approx 6,1$
 $\sin \alpha = \frac{a \sin \gamma}{c} = \frac{4 \cdot \frac{1}{2}}{6,1} = 0,3279 \Rightarrow \alpha \approx 19^\circ$
 $\alpha = 180^\circ - (\beta + \gamma) = 180^\circ - 65^\circ = 115^\circ$
 $\beta = 180^\circ - (29 + 30) = 121^\circ$
 $2R = \frac{c}{\sin \gamma} = \frac{6,1}{0,5} = 12,2 \Rightarrow R \approx 6,1$
 $S = \frac{1}{2} a \cdot b \cdot \frac{1}{2} = 7$

③ $a=3, b=5, c=6 \parallel \alpha=? \beta=? \gamma=? S=? R=?$



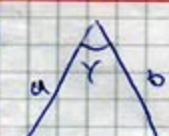
$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc} = \frac{25 + 36 - 9}{60} = \frac{52}{60} = 0,8666 \Rightarrow \alpha \approx 30^\circ$
 $\cos \beta = \frac{a^2 + c^2 - b^2}{2ac} = \frac{9 + 36 - 25}{36} = \frac{20}{36} = 0,5555 \Rightarrow \beta \approx 56^\circ$
 $\gamma = 180^\circ - 86^\circ = 94^\circ$
 $2R = \frac{3}{\sin 30^\circ} = 6 \Rightarrow R = 3$
 $S = \frac{1}{2} b \cdot c \cdot \sin \alpha = \frac{1}{2} \cdot 5 \cdot 6 \cdot \frac{1}{2} = 7,5$

④ $a=1, b=8, \gamma=40^\circ \parallel c=? \beta=? \alpha=? S=? R=?$




$c^2 = 1 + 64 - 16 \cdot 0,7660 = 4,3$
 $\sin \beta = \frac{b \sin \gamma}{c} = \frac{8 \cdot 0,6428}{2,07} = 0,2471 \Rightarrow \beta \approx 14^\circ$
 $\alpha = 180^\circ - (33 + 40) = 107^\circ$
 $2R = \frac{c}{\sin \gamma} = \frac{2,07}{0,6428} \approx 3,2$
 $R \approx 1,6$
 $S = \frac{1}{2} a \cdot b \cdot \sin \gamma = \frac{1}{2} \cdot 1 \cdot 8 \cdot 0,6428 = 2,6$

⑤ Enqashur me 3. $a=6, b=8, c=10 \parallel \alpha=? \beta=? \gamma=? S=? R=?$



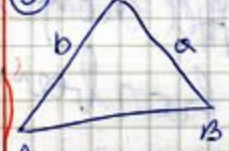
$a=b=c=5 \Rightarrow \alpha=\beta=\gamma=60^\circ \Rightarrow a=b=c=5$
 $S = \frac{a^2 \sqrt{3}}{4} = \frac{25\sqrt{3}}{4} \approx 10,8$
 $R = \frac{abc}{4S} = \frac{5^3}{4 \cdot \frac{25\sqrt{3}}{4}} = \frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$
 $R \approx 2,9$

⑦ $\alpha=30^\circ, R=2, \beta=40^\circ \parallel a=? b=? c=? S=?$




$\gamma = 180^\circ - (40^\circ + 30^\circ) = 110^\circ$
 $b = 2R \cdot \sin \beta = 4 \cdot 0,6428 = 2,6$
 $\frac{a}{\sin \alpha} = 2R \Rightarrow a = 2R \cdot \sin \alpha = 4 \cdot \frac{1}{2} = 2$
 $c = 2R \cdot \sin \gamma = 4 \cdot \sin 70^\circ = 4 \cdot 0,9397 \approx 3,8$
 $S = \frac{1}{2} b \cdot c \cdot \sin \alpha = \frac{1}{2} \cdot 2,6 \cdot 3,8 \cdot \frac{1}{2} \approx 2,5$

⑧ $S=4, a=2, b=8 \parallel c=? R=? \alpha=? \beta=? \gamma=?$



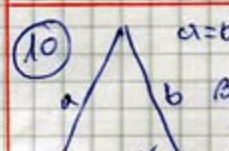
$S = \frac{1}{2} a \cdot b \cdot \sin \gamma \Rightarrow \sin \gamma = \frac{2S}{ab} = \frac{8}{16} = \frac{1}{2} \Rightarrow \gamma = 30^\circ$
 $c^2 = a^2 + b^2 - 2ab \cos \gamma = 4 + 64 - 2 \cdot 2 \cdot 8 \cdot \frac{\sqrt{3}}{2} = 52 \Rightarrow c \approx 7,2$
 $R = \frac{abc}{4S} = \frac{2 \cdot 8 \cdot 7,2}{16} = 7,2$
 $\frac{a}{\sin \alpha} = 2R \Rightarrow \sin \alpha = \frac{a}{2R} = \frac{2}{14,4} \approx 0,1388 \Rightarrow \alpha \approx 8^\circ$
 $\beta = 180^\circ - 38^\circ = 142^\circ$

⑨ $\alpha=45^\circ, \beta=75^\circ, c=5 \parallel \gamma=? a=? b=? S=? R=?$



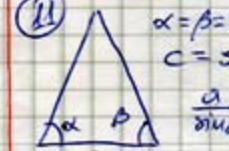
$\gamma = 180^\circ - (45^\circ + 75^\circ) = 60^\circ$
 $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} = 2R$
 $a = \frac{c \sin \alpha}{\sin \gamma} = \frac{5 \cdot \frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2}} = \frac{5\sqrt{6}}{3} \approx 4,1$
 $b = \frac{c \sin \beta}{\sin \gamma} = \frac{5 \cdot 0,9659}{0,8660} \approx 5,6$
 $2R = \frac{a}{\sin \alpha} = \frac{4,1}{0,7071} \approx 5,8 \Rightarrow R \approx 2,9$

⑩ $a=b=6, \alpha=30^\circ \parallel c=? \beta=? \gamma=? S=? R=?$




$\beta = \alpha = 30^\circ$
 $\gamma = 180^\circ - (30^\circ + 30^\circ) = 120^\circ$
 $c^2 = 36 + 36 - 2 \cdot 36 \cdot \cos 120^\circ = 72 + 72 = 144 \Rightarrow c = 12$
 $S = \frac{1}{2} a \cdot b \cdot \sin 120^\circ = 18 \cdot 0,8660 \approx 15,6$
 $R = \frac{abc}{4S} = \frac{6 \cdot 6 \cdot 12}{4 \cdot 15,6} = \frac{374,4}{62,4} = 6$

⑪ $\alpha=\beta=70^\circ \parallel a=? b=? \gamma=? S=? R=?$



$\gamma = 180^\circ - (70^\circ + 70^\circ) = 40^\circ$
 $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} = 2R$
 $a = \frac{c \sin \alpha}{\sin \gamma} = \frac{5 \cdot 0,9397}{0,6428} = 7,3$
 $b = a = 7,3$
 $2R = \frac{c}{\sin \gamma} = \frac{5}{0,6428} \approx 7,8 \Rightarrow R = 3,9$
 $S = \frac{1}{2} a \cdot b \cdot \sin 40^\circ = \frac{1}{2} \cdot 7,3 \cdot 7,3 \cdot 0,6428 \approx 17,1$

⑫ $a=\sqrt{3}, b=1, \gamma=150^\circ \parallel c=? S=? R=? \alpha=? \beta=?$



$S = \frac{1}{2} a \cdot b \cdot \sin 150^\circ = \frac{1}{2} \cdot \sqrt{3} \cdot 1 \cdot \frac{1}{2} = \frac{\sqrt{3}}{4}$
 $c^2 = a^2 + b^2 - 2ab \cos \gamma = 3 + 1 - 2 \cdot \sqrt{3} \cdot (-\cos 30^\circ) = 4 + 2 \cdot \sqrt{3} \cdot \frac{\sqrt{3}}{2} = 7$
 $c = \sqrt{7}$
 $\sin \alpha = \frac{a \sin \gamma}{c} = \frac{\sqrt{3} \cdot \frac{1}{2}}{\sqrt{7}} = \frac{\sqrt{21}}{2\sqrt{7}} = \frac{\sqrt{3}}{2} \approx 0,8660 \Rightarrow \alpha \approx 60^\circ$
 $\beta = 180^\circ - (19^\circ + 150^\circ) = 11^\circ$
 $2R = \frac{c}{\sin \alpha} = \frac{\sqrt{7}}{0,8660} = 2\sqrt{7} \Rightarrow R = \sqrt{7}$